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Stochastic Models For Enhancing Agility of Supply Chains

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STOCHASTIC MODELS FOR ENHANCING AGILITY OF SUPPLY CHAINS

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Abstract

This paper deals with the application of stochastic inventory model to the three-tier supply chain and verifying the values obtained by mathematical model with discrete event simulation. We investigate three-stage serial supply chain with stochastic demand and fixed replenishment lead-time. Inventory holding costs are charged at each stage, and each stage may incur a consumer backorder penalty cost charged by primary supplier to secondary supplier. The customer-demand follows Poisson distribution. We implement Base Stock model for inventory control at both suppliers. Computer simulation is then designed in such a way that it satisfies all the assumptions for mathematical model. Simulation is run to validate the results obtained from the mathematical model.

Keywords

Supply Chains, Stochastic Models, Base Stock Model, Discrete Event Simulation, Optimization

Introduction

Comparison of results from the base stock model and discrete event simulation for three-tier supply chain where the demand follows a Poisson distribution is the primary subject of this paper. We have considered a hypothetical company with three-tier supply chain. Base Stock Inventory Model is applied at the primary supplier, secondary supplier and at the warehouse. We calculated the fill rate, probability that the order has arrived before demand for each case and calculated reorder points at primary supplier, secondary supplier and warehouse for five replenishment lead times (12,8,6,4 and 2 months)(Table No 2) using this mathematical model. Discrete event simulation in ProModel is run to confirm the optimum inventory levels i.e. reorder points at warehouse, primary supplier and secondary supplier.

Background

Inventory management within the supply chain is critical when the demand is not deterministic. Demand variability increases as one move up the supply chain away from customer and any small changes in customer demand can result in large variation in orders upstream. This phenomenon is known as Bullwhip effect. Thus, it is necessary to study inventory models for uncertain demand. Wilson (1934) has done major work on statistical modeling of production and inventory control. Wilson breaks the inventory control problem into two distinct parts: 1. Determining the order quantity, which is the amount of inventory that will be produced with each replenishment. 2. Determining the reorder point or the inventory level at which replenishment will be triggered. P Zipkin (1999) emphasized on backorder policies in multistage supply chain where base stock inventory model is used. Wincel and Jeffrey P (2004) introduce lean methodology as the key factor in its supply chain strategies. Issues related to streamlining supply chain are discussed by Copacino, William C and Cooper (1999). Inventory issues in supply chain are explored further by Handfield, Robert B. (1999), Nichols, Ayers and James B. (1999).

Mathematical Models

Taylor's principles of scientific management (1903) were precursor to a host of mathematical models designed to solve the problems associated with manufacturing planning and control. These models formed the foundation for instruction in several operations management (OM) areas like inventory control, scheduling, capacity planning, forecasting and quality control. Of these areas, inventory control saw the development of a variety of mathematical models. These models can be subdivided into two broad areas. Those, that assumed demand to be known and those, which assumed demand to be stochastic in nature. Stochastic models are discussed in more detail here.

1. The Base Stock Model

The Base stock Model uses a continuous time frame and makes the following assumptions:

1. Demands occur one at a time.
2. Any demand not filled from stock is backordered.
3. Replenishment lead times are fixed and known.
4. Replenishments are ordered one at a time.
5. Products can be analyzed individually.

We make use of the following notations:
l = Replenishment lead time (in years)
x = Demand during replenishment lead time (in units), a random variable
G (x) = P (X<=x), cumulative distribution function of demand during replenishment lead-time; we will allow G to be

continuous or discrete.
$\theta = E[X]$ = mean demand (in units) during lead time l
h = cost to carry one unit of inventory for one year
b = cost to carry one unit of backorder for one year
r = reorder point which represents the inventory level that triggers a replenishment order
$R = r + 1$ base stock level
$S = r - \theta$, safety stock level
The fraction of demands filled from stock (as opposed to backordered), which we call the service level or fill rate.

As the order is placed every time a demand occurs, the relationship
Inventory + orders = R

The probability that the order arrives before its demand (i.e. does not result in a backorder) is given by $P(X<R)$.
The fraction of demands that are filled from stock is equal to the probability that an order arrives before the demand it has occurred.

$P(X<R) = G(R)$ if demand is continuous
$G(r)$ if demand is discrete

Hence $G(R)$, $G(r)$ represents the fraction of demands that will be filled from stock (i.e. fill rate). Base stock model is equivalent to the Japanese Kanban System (with kanban size of one) since, order quantity is one

The primary insights from the model:

1. Reorder points control the probability of stockouts by establishing safety stock.
2. To achieve a given fill rate, the required base stock level (and hence safety stock) will be an increasing function of both mean and standard deviation of the demand during replenishment lead time.
3. Base stock levels in multistage production systems are very similar to kanban.

We have assumed Poisson distribution for demand and found out reorder point, order quantity and the safety stock in supply chain.

2. Application Runs of Base Stock Model to Three-Tier Supply Chain

Replenishment lead time = 12 months

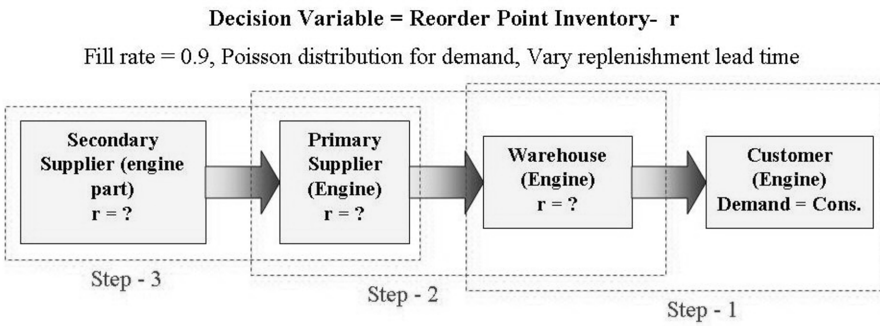

Figure 1. Supply chain considered for Base Stock model

At Warehouse, the demand during 12 months is 10 units /year
Average Demand = 10 units per year
P (k) = Probability (Demand during lead time, k)

$$= \frac{\theta^k e^{-\theta}}{k!} = \frac{10^k e^{-10}}{k!} \; ; \; G(r) = \sum_{k=0}^{r} p(k)$$

r	P(r)	G(r)
0	0.00	0.00
1	0.00	0.00
2	0.00	0.00
3	0.01	0.01
4	0.02	0.03
5	0.04	0.07
6	0.06	0.13
7	0.09	0.22
8	0.11	0.33
9	0.13	0.46

Table 1: Fill rate for various values of r

If the customer has an average demand of 10 units (say engines) per year then, for a fill rate of 90%, we see from Table 1, that the value of reorder point, r = 14 units per year at warehouse. Similarly we identify reorder point, r at Primary Supplier and Secondary Supplier for various replenishment lead time of 8, 6, 4 and 2 months. (Table 2)

3. Results from Base Stock Model

Table 2 summarizes all the results for base stock model and frequency of order. Order cost is assumed to be $ 25 per order. The total cost is calculated by using

$$TC = c\left(\frac{Q}{2} + r - \theta\right) + \text{Order cost.}$$

3a. Total Cost VS. Replenishment Lead-time

The total inventory cost is plotted against replenishment lead time in Figure 2.

Table 2
Summary of Results of Costs (Base Stock Model)

Replenishment Lead Time (months)	Warehouse ($)	Primary Supplier ($)	Secondary Supplier ($)
12	925	1175	1450
8	741.25	925	1175
6	775	925	1225
4	725.5	975	1350
2	316.25	450	650

Table 3

Summary of Application Runs of Base Stock Model

Replenishment Lead Time	Demand	Reorder Point(r)	Q	Location	Frequency of order (F=D/Q)	Average Demand	Order Cost	Total Cost
12	10	14.00	1.00	Warehouse	10.00	10	250	925
	14	19.00		PS	14.00	14	350	1175
	19	25.00		SS	19.00	19	475	1450
8	6.67	10.00	1.00	Warehouse	6.67	6.67	166.75	741.25
	10	14.00		PS	10.00	10	250	925
	14	19.00		SS	14.00	14	350	1175
6	10	8.00	1.00	Warehouse	10.00	5	250	775
	16	11.00		PS	16.00	8	400	925
	22	15.00		SS	22.00	11	550	1225
4	10	6.00	1.00	Warehouse	10.00	3.33	250	725.5
	18	9.00		PS	18.00	6	450	975
	27	13.00		SS	27.00	9	675	1350
2	1.67	3.00	1.00	Warehouse	1.67	1.67	41.75	316.25
	3	5.00		PS	3.00	3	75	450
	5	8.00		SS	5.00	3	125	950

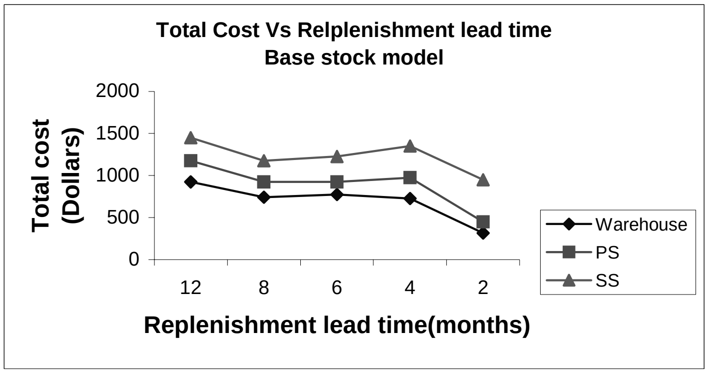

Figure 2: Total Cost vs. Replenishment Lead- time (Base Stock Model)

3b. Reorder Point vs. Replenishment Lead time

The reorder point decreases with replenishment lead- time. Reorder point is plotted against replenishment lead time in Figure 3.

Replenishment lead time (months)	Reorder point at warehouse	Reorder point at primary supplier	Reorder point at secondary supplier
12	14	19	25
8	10	14	19
6	8	11	15
4	6	9	13
2	3	5	8

Table 4: Reorder Point for Base Stock Model

4. Summary of Base Stock Model

The graph in Figure 3 shows the decreasing trend in reorder point from warehouse to secondary supplier for the same lead-time. The total inventory cost decreases with replenishment lead-time for Base Stock Model. We can conclude from Figure 2 that there is decreasing trend in costs of warehouse, primary supplier and secondary supplier for the same replenishment lead-time.

Base stock model emphasizes on order quantity of 1 and the model can be used where demand is stochastic. Base stock model proves to be better for small lead-time.

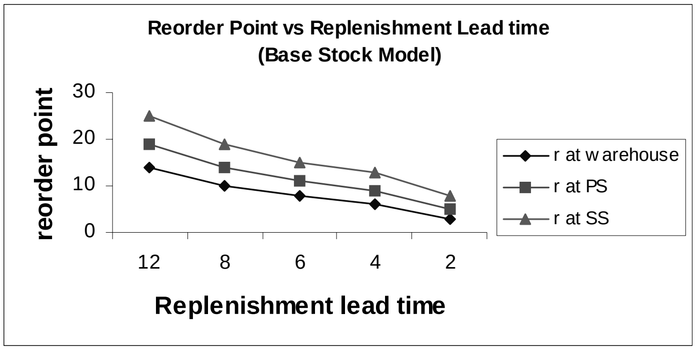

Figure 3: Reorder Point vs. Replenishment Lead-time (Base Stock Model)

Discrete Event Simulation Model

Computer based simulation is the "*imitation of a dynamic system using a computer model in order to evaluate and improve system performance*". Harrell, Charles R. (2004). In practice, simulation is usually performed using commercial simulation software like ProModel that have modeling constructs specifically designed for capturing the dynamic behavior of system. Performance statistics are gathered during the simulation and automatically summarized for analysis. Modern simulation software provides a realistic, graphical animation of the system being modeled. During the simulation, the user can interactively adjust the speed and model parameter values to do a "what if" analysis. Some simulation software provides optimization technology also. Trial and error approaches are expensive, time consuming and disruptive. The power of simulation lies in the fact that it provides a method of analysis that is not only formal and predictive, but is capable of accurately predicting the performance of even the most complex systems.

A discrete event simulation model is created using ProModel software to assess the performance of a two-tier supply chain. Base stock Model and (Q, r) Model were applied to this supply chain in the previous sections.

1. Goals of Computer Based Simulation

Primary goal of this computer-based simulation is to demonstrate that Base Stock Model can effectively predict the level of inventory at reorder point. Another goal is to compare the results obtained here with those of mathematical model and physical simulation model.

2. Simulation Layout

Discrete event simulation is a pedagogical tool that uses computer models to study a production system with the goal of optimizing its performance. ProModel simulation software is used for analyzing and assessing the flow of parts through a two-tier supply chain system. A computer model of a two-tier supply chain was build using ProModel software. The model uses four locations to indicate the key players in the supply chain namely Customer, Ware House, primary Supplier and Secondary Supplier. The layout of the model is shown in Figure 4.

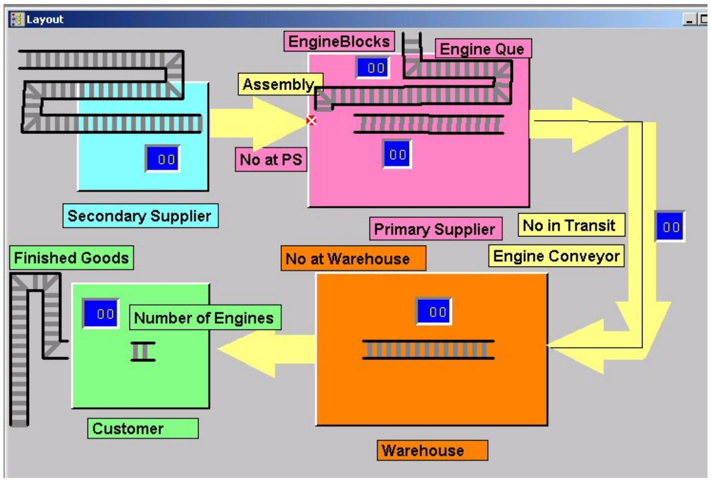

Figure 4. Layout of the Supply Chain in ProModel

The model uses real time counters and global variables to define and display the number of parts as they go through the supply chain. The conveyors are designed long enough to display all parts as they are waiting to be processed. A specified number of cylinders arrive at the secondary supplier with a Poisson distribution. Engine blocks arrive at the primary supplier with another Poisson distribution. One cylinder is assembled with the engine block at the assembly station. Engine block icon is initially gray in color. After assembly of cylinder, the color of the engine block changes to blue indicating an assembled engine. The assembled engine proceeds to the warehouse via engine conveyor and then on to customer. The replenishment lead-time is simulated by the travel delay between these stations. For example, if the replenishment lead-time is 2 months, transportation between these stations takes 2 months.

3. Simulation Results

The goal behind building the computer based simulation model is to see if the results produced by the mathematical models can be replicated. This can be done easily by first running the simulation without any inventory in the supply chain. This will produce stock outs and backorders. If we then run the model with the inventory positions predicted by the base stock model and can show that customer demands are met without any backorders that will be an indication that the results from mathematical models have been validated. The simulation model was run first with no inventory positions in the supply chain. The screen display for this case is produced in Figure 5. The counter located at the customer box (green) indicates the total number of engines delivered to the customer. In this case, seven engines were delivered to customer with three backorders. The mean demand is assumed to be the same as in previous runs of mathematical models, i.e. 10.

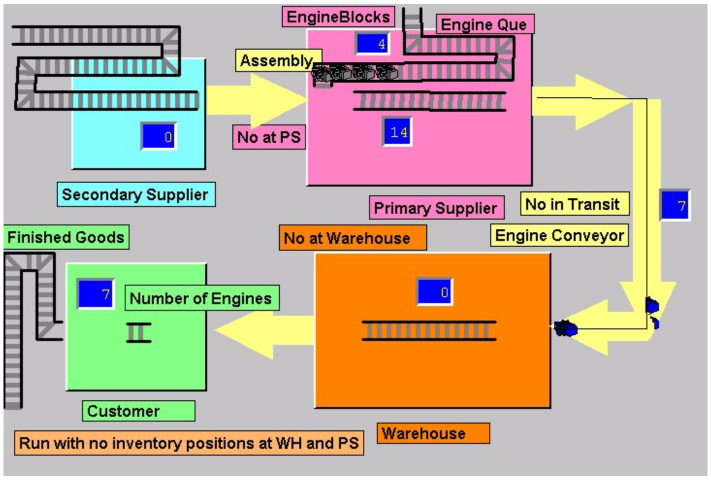

Figure 5. Screen Display for Case with Zero Inventory Positions

Next we run the simulation with the values of r predicted by the base stock model. For example, the base stock model predicted that to obtain a fill rate of 90%, following inventory levels must be maintained; warehouse-3, primary supplier-5 and secondary supplier-8 for a customer demand of 10 units/yr and replenishment lead time of 2 months. The part counter in this case indicates that 10 engines were delivered to the customer without any backorder. These results are summarized in Table 5.

Case	Inventory at PS	Inventory at WH	Lead time	Engines to Customer	Number of Backorders
1	0	0	60 days	7	3
2	5	3	60 days	10	0

Table 5. Results from ProModel

Table 5 shows the inventory levels and number of engines produced during the two cases for lead-time of 60 days. Customer demands are met with no backorders when predicted values of inventory position are used.

Conclusions

Discrete event simulation is designed to include all the assumptions made by mathematical model. Hence, Base Stock Model and computer simulation models are comparable. Computer simulations demonstrate that customer demands are met with no backorders when predicted values of inventory position are used from the Base Stock Model. It is also shown that if zero initial inventory positions are used stockout and backorder occurs.

Base Stock Model is effective when the demand is not deterministic and service factor assumed in mathematical model is 0.9, which is quite acceptable. Base stock model assumes replenishment order quantity as 1 and the total inventory cost decreases with replenishment lead time. Base stock model is beneficial for supply chains having short replenishment lead time. Discrete event simulation model validates the results obtained from the Base Stock Model.

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